# Hale School

## Mathematics Specialist

## Test 1 --- Term 1 2016

## Complex Numbers

**Name:** ______________________________

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| / 43 |

**Instructions:**

- **CAS calculators are NOT allowed**
- **External notes are not allowed**
- **Duration of test: 45 minutes**
- **Show your working clearly**
- **Use the method specified (if any) in the question to show your working (Otherwise, no marks awarded)**
- **This test contributes to 7% of the year (school) mark**

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All arguments must be given using principal values.

Question 1 (4 marks: 1, 1, 1, 1)

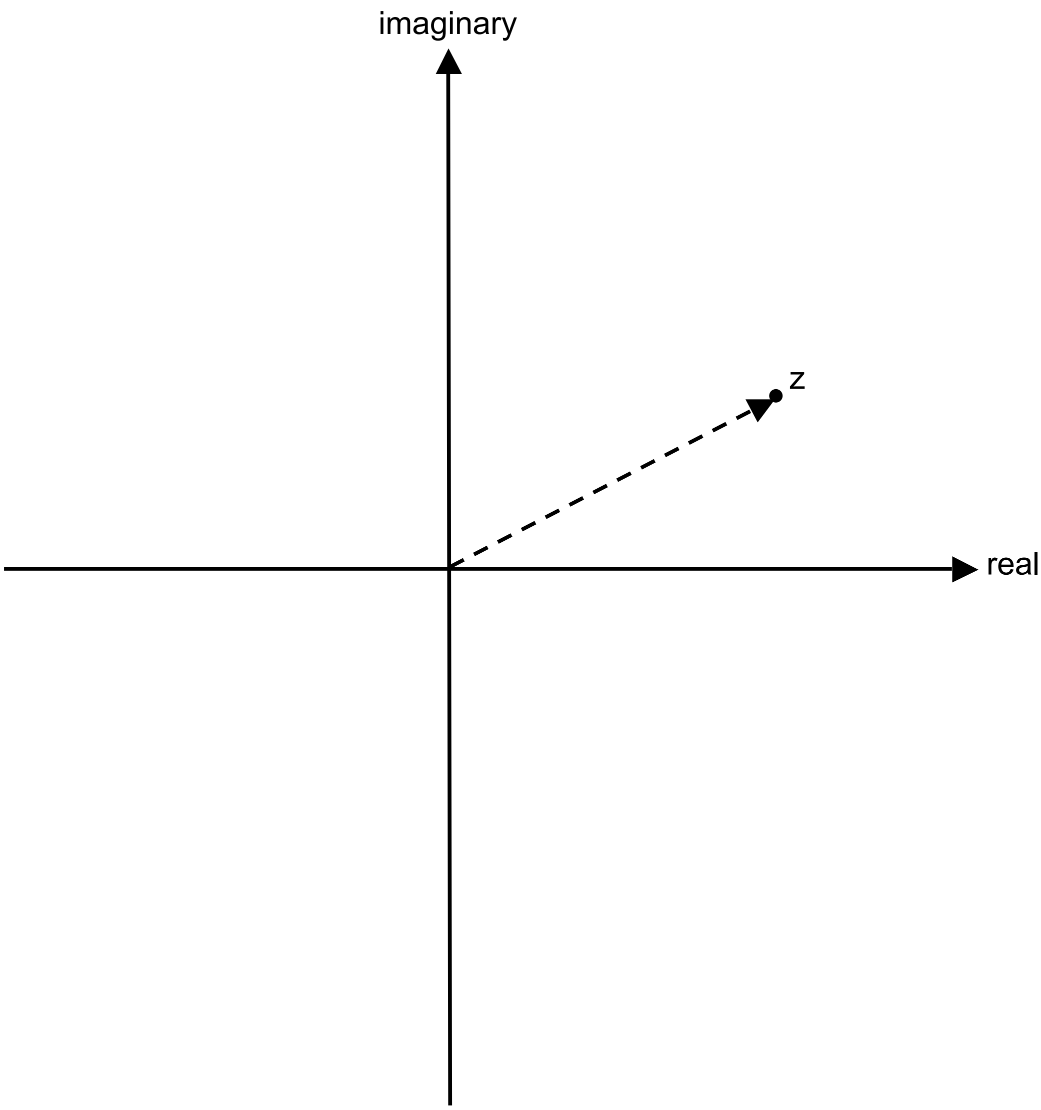
The following diagram shows a complex number $z$ on the complex plane. Locate the following complex numbers. Label your answers clearly.

(a) $z_1 = \bar{z}$ (1 mark)

(b) $z_2 = \frac{1}{z}$ given that $|z|^2 = 2$ (1 mark)

(c) $z_3 = iz$ (1 mark)

(d) $z_4 = \frac{z}{i}$ (1 mark)



Question 2 (9 marks: 2, 3, 4)

(a) Convert $z = -1 + i$ to polar form. (2 marks)

(b) Determine the value(s) of $\theta$ and $x$ if $z = 6 \text{ cis } \theta = x - 3i$. (3 marks)

(c) Determine the argument of $z = \dfrac{1}{\sin\frac{7\pi}{10} - i\cos\frac{7\pi}{10}}$. (4 marks)

Question 3 (8 marks: 4, 4)

(a) **Given** that if $z = \cos\theta + i\sin\theta$ then $z^n = \cos(n\theta) + i\sin(n\theta)$ is true for all positive integers.

Show that $z^n = \cos(n\theta) + i\sin(n\theta)$ is also true for all **negative** integers. (4 marks)

(b) Use De Moivre's theorem to show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. (4 marks)

Question 4 (7 marks: 2, 2, 3)

It is given that **one** of the roots of the complex equation $z^3 = u$ is $2 \text{ cis } \frac{2\pi}{5}$.

(a) Write down all the other roots in polar form. (2 marks)

(b) Determine the complex number u. (2 marks)

(c) Use the given information that $2 \text{ cis } \frac{2\pi}{5}$ is a solution of $z^3 = u$ to determine **one** solution of the complex equation $z^3 = -ui$. (3 marks)

Question 5 (5 marks: 2, 3)

(a) The locus of $z$ such that $| z - (1 + i) | = | z - (2 - 4i) |$ can be interpreted geometrically as the set of points equidistant from $(1, 1)$ and $(2, -4)$.

Give a similar geometrical interpretation of $| z - (1 + i) | = | 2z - (2 - 4i) |$. (2 marks)

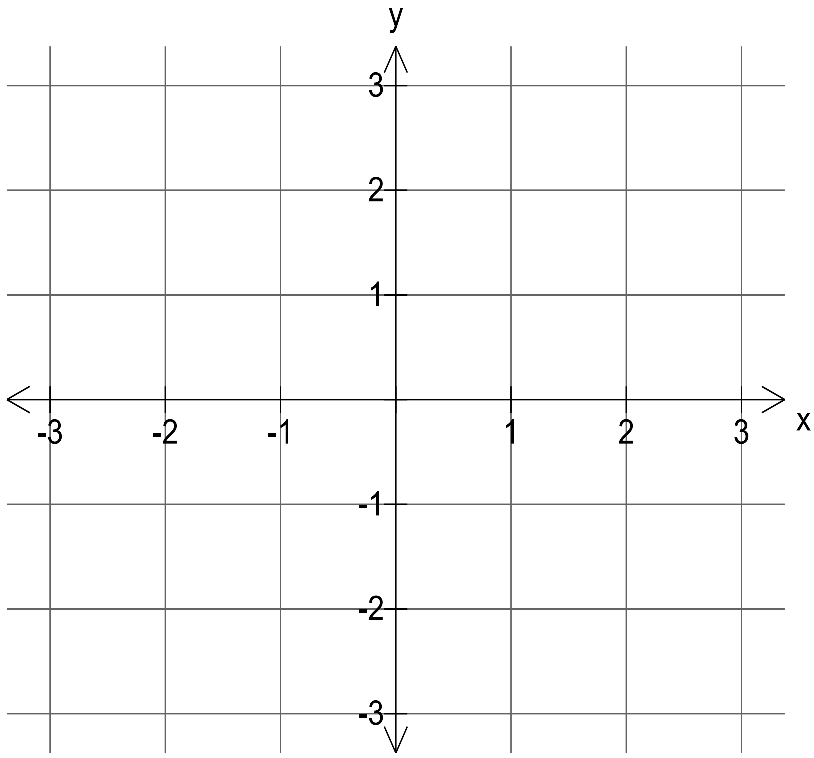
(b) Determine the Cartesian equation of $| z - (1 + i) | = | 2z - (2 - 4i) |$. (3 marks)

Question 6 (6 marks: 4, 2)

(a) Sketch the locus of u, v and w if

(i) $\arg(u) = -\frac{3\pi}{4}$ (ii) $\arg(v+1) = -\frac{3\pi}{4}$ (iii) $\arg(w+1-2i) = -\frac{3\pi}{4}$

Label the loci clearly. (4 marks)




(b) Determine z such that $\arg(z+1-2i) = -\frac{3\pi}{4}$ and $|z|$ is a minimum. (2 marks)

Question 7 (4 marks: 2, 2)

(a) Show that $(z - i)$ is a factor of $P(z) = z^3 + iz^2 + (2 - 7i)z - 7$. (2 marks)

(b) Form a polynomial with **integer** coefficients such that $x = \sqrt{2}$ and $x = i$ are two of the roots of the polynomial. (2 marks)
(Note: The polynomial has other roots.)